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SUBJECT: Sensitivity of Planetary Parking Orbits to Uncertainties in the Mass and Oblateness Parameters Case 105-4

DATE: August 12, 1970

FROM: H. B. Bosch

ABSTRACT

The oblate gravity potential of a planet can be utilized to bring a parking orbit, which is initially aligned with a given arrival trajectory, into alignment with a desired departure trajectory at the end of a specified time interval. For planetary missions with long staytimes, this technique can eliminate the velocity requirements for effecting the orientation change between arrival and departure directions. It is shown that the actual hyperbolic excess velocity vector will deviate from the nominal one depending on the accuracy to which the mass parameter (μ) and oblateness parameter ($\sigma = J_2R^2$) are known. The relative uncertainties in these parameters for Mars are $d\mu/\mu = \pm 4.67 \times 10^{-6}$ and $d\sigma/\sigma = 2.29 \times 10^{-3}$. Incremental velocities for compensating for these uncertainties are calculated for a 1986 Mars conjunction mission with a 580 day staytime. These velocities range between 40 and 365 fps. The corresponding contingency propellant requirements are approximately 0.3% and 3% of the total weight of the orbiting spacecraft.

(NASA-CR-113380) SENSITIVITY OF PLANETARY PARKING ORBITS TO UNCERTAINTIES IN THE MASS AND OBLATENESS PARAMETERS (Bellcomm, Inc.) 21 p

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MEMORANDUM FOR FILE

I. INTRODUCTION

Thibodeau (Ref. 1) examined a technique whereby the effects of a planet's oblateness can be utilized so as to accomplish the required orientation change between the approach and departure asymptotes for planetary missions with long stay-times. He showed that an inclination and eccentricity can be predetermined for a parking orbit which will be aligned with the given approach trajectory at periapsis, and then experience appropriate nodal and apsidal rotations so as to be aligned with the desired departure trajectory at periapsis, all in a fixed time interval.

The success of such a technique depends on the accuracy of knowledge of the mass and shape (oblateness) parameters of the planet. This memorandum will determine the sensitivity of the resulting departure direction to such uncertainties, and the propellant requirements for corrective velocity impulses.

II. THE NOMINAL PARKING ORBIT

Let the magnitude and direction of the approach hyperbolic excess velocity vector be given, as well as the desired magnitude and direction of the departure hyperbolic excess velocity vector. In addition let the periapsis radius of the parking orbit and the staytime be given mission parameters.

The orientation of the parking orbit at the time of arrival can be calculated from*

$$\Omega_A = \alpha_A - \arcsin\left(\frac{\tan\delta_A}{\tan i}\right) \qquad (1)$$

*See List of Symbols and Figures 1 and 2.

$$\omega_A = \arcsin\left(\frac{\sin\ \delta_A}{\sin\ i}\right) - \theta_A \tag{2}$$

where

$$\cos\theta_A = \frac{1}{1 + V_A^2\ r_p/\mu} \tag{3}$$

Similarly the orientation at departure can be calculated from

$$\Omega_D = \alpha_D - \arcsin\left(\frac{\tan\ \delta_D}{\tan\ i}\right) \tag{4}$$

$$\omega_D = \arcsin\left(\frac{\sin\ \delta_D}{\sin\ i}\right) - \theta_D \tag{5}$$

where

$$\cos\theta_D = \frac{-1}{1 + V_D^2\ r_p/\mu} \tag{6}$$

The assumption here is that the nonspherical potential of the planet affects the ascending node (Ω) and argument of periapsis (ω) but not the inclination (i) or periapsis radius (r_p).

Thus, for any selected inclination, the required nodal and apsidal rotations can be calculated from

$$\Delta\Omega = \Omega_D - \Omega_A$$

$$\Delta\omega = \omega_D - \omega_A$$

In particular, the inclination can be determined from the requirement that both of the above rotations must be accomplished in the same time interval; i.e.

$$\frac{\Delta\Omega}{\dot{\Omega}} = \frac{\Delta\omega}{\dot{\omega}} \tag{7}$$

Since the respective rotation rates are given by

$$\dot{\Omega} = -\frac{3}{2}\sqrt{\mu}\,\sigma\, r_p^{-7/2}\,(1-e)^{3/2}\,(1+e)^{-2}\cos i \tag{8}$$

$$\dot{\omega} = -\frac{3}{2}\sqrt{\mu}\,\sigma\, r_p^{-7/2}\,(1-e)^{3/2}\,(1+e)^{-2}\left(\frac{5}{2}\sin^2 i-2\right) \tag{9}$$

this means that the required inclination emerges as a solution of the transcendental equation

$$\frac{\Delta\Omega(i)}{\Delta\omega(i)} = \frac{2\cos i}{5\sin^2 i-4} \tag{10}$$

Values for i can thus be determined by using equations (1), (2), (4), (5), and (10). Numerical values for $\Delta\Omega$ can then be calculated by use of (4) and (1). Now the requirement that the nodal rotation must take place specifically in a time interval of length T, i.e.

$$\dot{\Omega}(e) = \Delta\Omega/T \tag{11}$$

gives an equation from which the required eccentricity can be calculated.*

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*Note that the eccentricity can just as well be calculated by considering the apsidal instead of nodal rotation rate.

The result of these computations is that a parking orbit with the parameters i, e, r_p, Ω_A, ω_A will rotate into an orbit with the parameters i, e, r_p, Ω_D, ω_D in a time interval of length T. This is the essence of the technique described by Thibodeau (Ref. 1).

III. EFFECTS OF UNCERTAINTIES

Assuming the planet's mass (μ) and shape (σ) parameters to be known exactly, a nominal sequence of maneuvers might be as follows: at a predetermined point along the approach trajectory the speed and direction are adjusted so as to place the spacecraft onto a precalculated planetocentric hyperbola. Upon arrival at periapsis a retrothrust is given to place the spacecraft onto an ellipse of the proper eccentricity. After a time interval of length T the orbit will be so aligned that another precalculated, collinear thrust at periapsis puts the spacecraft onto a hyperbola which again has the desired asymptote. The spacecraft should now be on the desired heliocentric trajectory.*

However our knowledge of the numerical values of μ and σ is imperfect. Thus the hyperbola on which the spacecraft approaches the planet will be such that the argument and radius of periapsis will differ from the nominal values because the actual planetary mass will be different from the nominal mass. Therefore, when the nominal retrothrust is applied, the resulting elliptic eccentricity will not be nominal either. Then, because the actual oblateness differs from the nominal, the orbital alignment at departure time will not be what it was predicted to be. Now the nominal collinear thrust at periapsis will place the spacecraft onto a departure hyperbola which will also not be the desired trajectory.

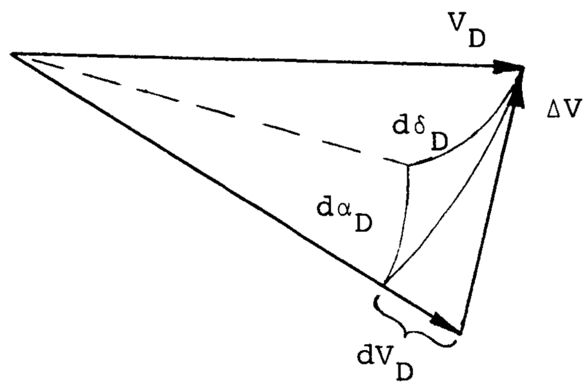
Formulas are presented in Appendix A by which the sensitivities of these orbital parameters to the relative errors $d\mu/\mu$ and $d\sigma/\sigma$ can be calculated.

IV. A CORRECTIVE MANEUVER

As a result of executing only the nominal retrothrust at arrival and the nominal collinear thrust at departure, the spacecraft is now departing on the "wrong"

*Navigation and execution errors will not be considered in this memorandum.

trajectory. Sufficiently far from the planet (such as, at the "sphere of influence") the actual velocity vector will differ from the nominal asymptotic velocity vector in magnitude, right ascension and declination by the amounts dV_D, $d\alpha_D$, and $d\delta_D$, respectively.



From the accompanying sketch, the magnitude of a corrective velocity impulse can be calculated to be

$$\Delta V = V_D \sqrt{2\left(1+\frac{dV_D}{V_D}\right)\left(1-\cos\, d\alpha_D \cos\, d\delta_D\right) + \left(\frac{dV_D}{V_D}\right)^2} \qquad (12)$$

Two comments are in order here: First, this "corrected" velocity vector is only parallel to the desired vector V_D. However, as numerical calculations will bear out, this parallel displacement is ignorable, especially in the context of the heliocentric trajectory. Second, such an impulsive velocity change is not necessarily optimal. In fact, as a result of tracking and orbit determination, the actual error will be known before departure and a non-nominal escape burn will be executed, possibly off-periapsis. The intent of this study is not to develop an optimum strategy, but rather to estimate a quantitative bound on the effect that variations in μ and σ could have on a planetary mission.

V. NUMERICAL EXAMPLE FOR A 1986 MARS MISSION

The current best estimates of the mass and shape parameters for Mars are given by JPL (personal communication with D. L. Cain) as

$$\mu = (42,\ 828.32 \pm .2)\ \text{km}^3/\text{sec}^2$$

$$\sigma = (34,\ 109.3 \pm 78.)\ \text{km}^2$$

Treating the uncertainties as actual errors, calculations were made for a 1986 Mars conjunction mission using

$$\frac{d\mu}{\mu} = \frac{\pm .2}{42,828.32} = \pm\ 4.67 \times 10^{-6}$$

$$\frac{d\sigma}{\sigma} = \frac{\pm 78}{34,109.3} = \pm\ 2.29 \times 10^{-3}$$

Mission parameters were taken from Reference 2 as follows:

arrival asymptotic velocity:	V_A = 11,255 fps
	α_A = 358°.15
	δ_A = 22°.42
departure asymptotic velocity:	V_D = 12,516 fps
	α_D = 300°.75
	δ_D = 2°.51

Periapsis altitude is chosen to be 200 nm and staytime is T = 580 days.

Eight cases were selected from those generated by VanderVeen* (Ref. 2). These are identified in Table I by inclination (i) and orbital period (P) for comparison with Ref. 2. Also listed are the resulting variations in the departure asymptotic velocity vector and the magnitudes

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*It is shown in Appendix B that there are 64 candidate inclinations to be obtained from equation (10). Some of these may have to be rejected because they make equation (11) unsolvable.

of the required corrective impulses, as calculated in Appendix A. The values shown in Table I correspond to $d\mu$ and $d\sigma$ having the same sign. In all cases, calculations with $d\mu$ and $d\sigma$ having opposite signs resulted in numerically smaller variations. It appears from this list that eccentricity is the dominant factor affecting the magnitude of the corrective velocity impulse, in the sense that ΔV increases with decreasing e. However, a search for a continuous relationship between ΔV, e, and I would be meaningless because only a finite number of discrete parking orbits is possible (See Appendix B).

VI. SUMMARY

The sensitivity of the precessions of planetary parking orbits to uncertainties in the values of the planet's mass and oblateness parameters has been determined. Navigation and maneuver execution errors, as well as other perturbations, were not considered in this study. A (not necessarily optimum) one-impulse maneuver to compensate for the above uncertainties was considered, and the corresponding velocity requirements were thus determined.

Considering the current precision of our knowledge of these two parameters for Mars, these velocities are shown to vary between 40 fps and 365 fps for various parking orbits for a 1986 Mars conjunction mission with 580 days staytime. Assuming a specific impulse somewhere in the range 350 to 450 seconds, the ratio of required propellant to spacecraft total weight is 0.0028 to 0.0036 for 40 fps, and 0.025 to 0.032 for 365 fps.

In the technique described in this memorandum, the parking orbit is selected from a discrete set of (64 or fewer) orbits for which the nodal and apsidal rotation rates satisfy certain relationships. Other orbit selection criteria, such as altitude, frequency of overflight of selected regions, range of planetocentric latitude, etc., are therefore constrained.

H B Bosch

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Attachments
- List of Symbols
- References
- Table I
- Figures 1 and 2
- Appendixes A and B

ACKNOWLEDGEMENT

I am grateful to A. A. VanderVeen for the many discussions I had with him during the course of this study.

LIST OF SYMBOLS

b	-	approach offset
d	-	differential (representing a small change)
e	-	orbit eccentricity
G	-	universal gravitational constant
i	-	orbit inclination relative to planet equator
J_2	-	second coefficient in spherical harmonic expansion of gravitational potential
m	-	planet mass
P	-	orbit period
r_p	-	periapse radius
R	-	mean equatorial radius of planet
T	-	staytime
V	-	asymptotic velocity
V_o	-	circular velocity at radius r_p
α	-	planetocentric right ascension of asymptote
δ	-	planetocentric declination of asymptote
ΔV	-	magnitude of corrective velocity impulse
$\Delta\omega$	-	required apsidal rotation
$\Delta\Omega$	-	required nodal rotation
θ	-	planetocentric angle between periapse and asymptote
μ	-	mass parameter: Gm
σ	-	shape parameter: J_2R^2
ω	-	argument of periapse
Ω	-	right ascension of ascending node

List of Symbols Continued

$\dot{\omega}$ - apsidal rotation rate

$\dot{\Omega}$ - nodal rotation rate

Subscripts

A - conditions at arrival (approach)

D - conditions at departure

REFERENCES

1. Thibodeau, J. R. III, "Use of Planetary Oblateness for Parking Orbit Alignment," NASA TN D-4657, July, 1968.

2. VanderVeen, A. A., Computer Run of Program OBLATE, Bellcomm, Inc., January 12, 1970

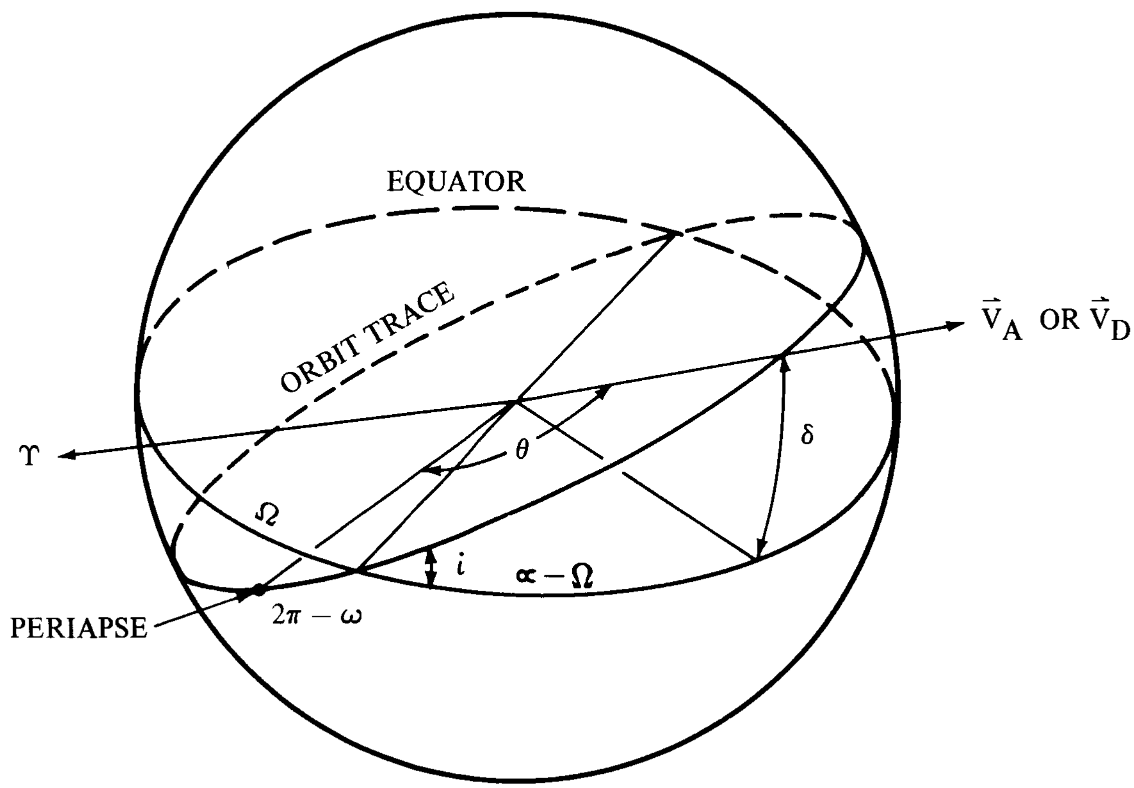

FIGURE 1 - ORIENTATION OF PARKING ORBIT PLANE RELATIVE TO HYPERBOLIC EXCESS VELOCITY VECTOR

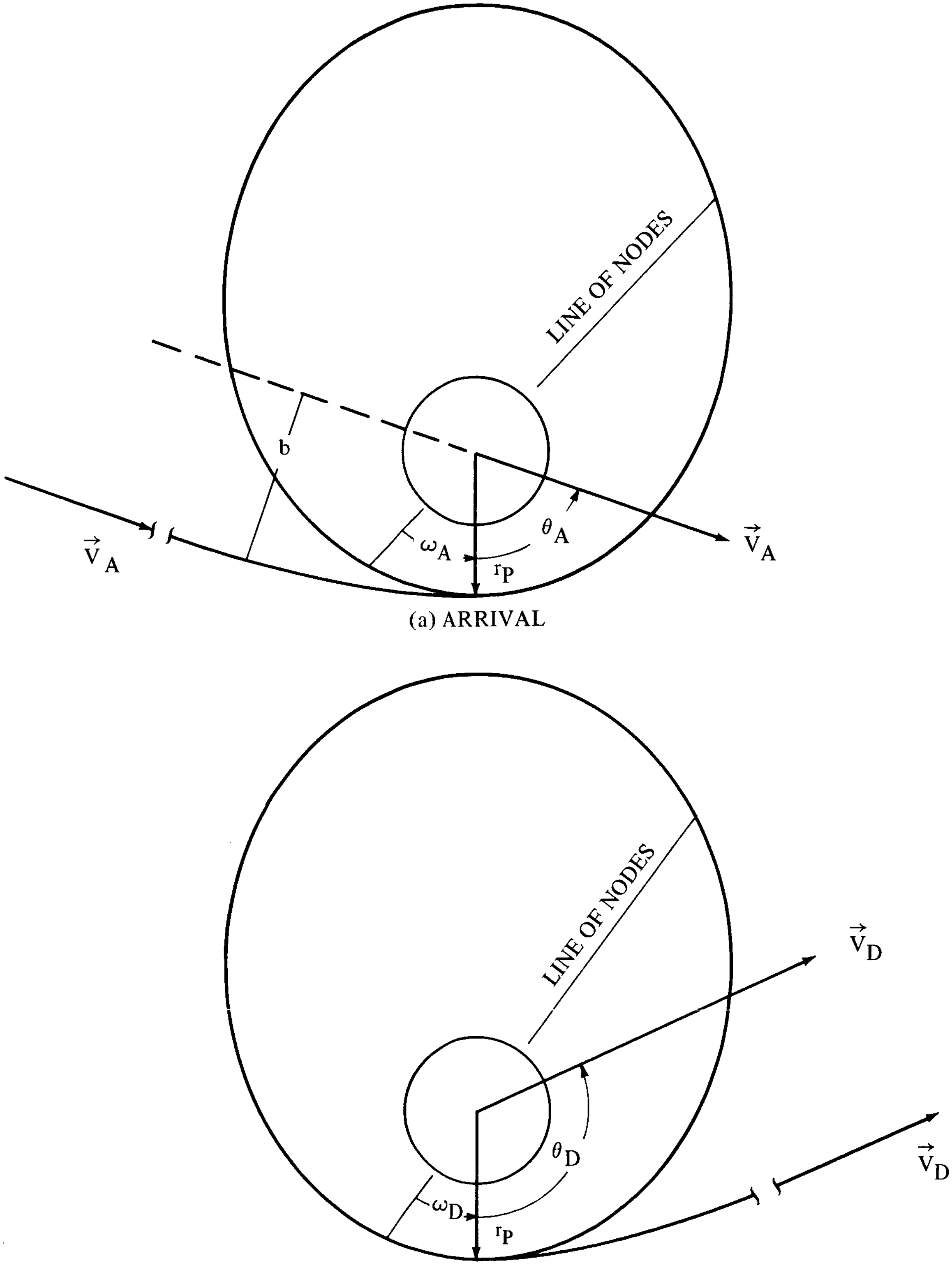

FIGURE 2 - INPLANE RELATIONSHIP BETWEEN PARKING ORBIT AND ARRIVAL AND DEPARTURE TRAJECTORIES

TABLE I. Perturbations and Corrective Velocity Requirements for for Eight Selected Parking Orbits for a 1986 Mars Conjunction Mission

i (deg)	P (hrs)	e	dV_D (fps)	$d\alpha_D$ (deg)	$d\delta_D$ (deg)	ΔV (fps)
69.2	20.4	0.79	0.003	-0.18	-0.07	42
135.3	18.2	0.77		0.08	0.26	59
133.2	10.4	0.67		0.21	0.47	112
27.1	10.4	0.67		0.41	0.64	166
86.4	5.9	0.52		-0.19	-1.00	222
64.4	5.5	0.49		-1.02	-0.07	223
115.9	4.1	0.39		1.58	-0.07	345
68.5	4.0	0.38		-1.57	-0.56	364

APPENDIX A

SENSITIVITY FORMULAS

1. Sensitivity of the Initial Orbit

The approach offset (see Fig. 2a) is given by

$$b = r_p \sqrt{1 + 2\mu / \left(r_p V_A^2\right)}$$

Treating the precalculated number b as constant, the sensitivity of r_p to a variation in μ can be calculated by differentiation. Thus

$$\frac{dr_p}{r_p} = - \cos\theta_A \frac{d\mu}{\mu} \tag{A1}$$

where θ_A is given by (3).

The argument of periapsis at arrival is given by (2). Differentiating this and (3) gives

$$d\omega_A = \cos\theta_A \sin\theta_A \frac{d\mu}{\mu} \tag{A2}$$

The retroburn, which is executed on arrival at periapsis, has magnitude

$$\Delta V_A = \sqrt{V_A^2 + 2V_o^2} - V_o \sqrt{1+e}$$

where

$$V_o^2 = \mu/r_p.$$

Treating ΔV_A as constant, the differential of this expression yields

$$de = \left[2 \sqrt{\frac{1+e}{2+(V_A/V_o)^2}} - (1+e) \right] (1 + \cos\theta_A) \frac{d\mu}{\mu} \qquad \text{(A3)}$$

2. <u>Sensitivity of the Final Orbit</u>

From the relationship

$$\Omega_D = \Omega_A + \dot{\Omega}T$$

we get

$$d\Omega_D = d\dot{\Omega}\, T$$

since Ω_A is independent of μ, σ, r_p and e. Differentiating (8) and using the equivalence

$$\dot{\Omega}T = \Delta\Omega$$

gives

$$d\Omega_D = \frac{\Delta\Omega}{2} \left[(1+7 \cos\theta_A) \frac{d\mu}{\mu} + 2\frac{d\sigma}{\sigma} - \left(\frac{7-e}{1-e^2} \right) de \right] \qquad \text{(A4)}$$

In an analogous fashion we get

$$d\omega_D = \frac{\Delta\omega}{2}\left[(1+7\cos\theta_A)\frac{d\mu}{\mu} + 2\frac{d\sigma}{\sigma} - \left(\frac{7-e}{1-e^2}\right) de\right] + d\omega_A \tag{A5}$$

3. <u>Sensitivity of the Departure Asymptote</u>

The hyperbolic excess velocity is given by

$$V_D^2 = \left(V_o\sqrt{1+e} + \Delta V_D\right)^2 - 2V_o^2$$

where ΔV_D represents the nominal retrothrust at periapsis at departure time. Treating this as constant, differentiation eventually gives

$$\frac{dV_D}{V_D} = \left(\frac{V_o}{V_D}\right)^2\left(\sqrt{\frac{V_D^2 + 2V_o^2}{V_A^2 + 2V_o^2}} - 1\right)\left(\frac{V_A^2 + 2V_o^2}{V_A^2 + V_o^2}\right)\frac{d\mu}{\mu} \tag{A6}$$

Differentiation of (6) yields the auxiliary formula

$$d\theta_D = \frac{1 + \cos\theta_D}{-\tan\theta_D}\left[\left(1 + \cos\theta_A\right)\frac{d\mu}{\mu} - 2\frac{dV_D}{V_D}\right] \tag{A7}$$

From Figure 1,

$$\cos i = \tan(\alpha_D - \Omega_D)/\tan(\omega_D + \theta_D)$$

Differentiation yields

$$d\alpha_D = d\Omega_D + \frac{\cos i}{\cos^2 \delta_D} (d\omega_D + d\theta_D) \tag{A8}$$

Similarly, from Figure 1,

$$\sin i = \sin\delta_D/\sin(\omega_D + \theta_D)$$

so that, after differentiating,

$$d\delta_D = \sqrt{1 - \cos^2 i/\cos^2 \delta_D}\,(d\omega_D + d\omega_D) \tag{A9}$$

APPENDIX B

MULTIPLICITY OF PARKING ORBITS

As the spacecraft approaches (or departs) the target planet the only geometric condition required of the parking orbit is that its plane contain the approach (or departure) asymptote. This leaves the orbit inclination totally free and several conditions can be imposed, as follows:

Whatever the numerical value of the inclination may be, there are always two orbit planes of the same inclination, both containing the approach asymptote. The ascending node is in the near hemisphere for one of these and in the far hemisphere for the other (as seen from the approach direction). Thus there are two possible values for Ω_A (see equation (1)) which differ by 180°. In an analogous fashion there are two possible values for Ω_D (equation (4)), also differing by 180°.

Next, it is possible to specify whether the rotation from Ω_A to Ω_D should be accomplished in less than one revolution or more than one -- i.e. whether $\Delta\Omega = \Omega_D - \Omega_A$ or $\Delta\Omega = 360° + \Omega_D - \Omega_A$. The same thing can be specified for the motion of periapse -- i.e. whether $\Delta\omega = \omega_D - \omega_A$ or $\Delta\omega = 360° + \omega_D - \omega_A$. These choices may depend on the desirability of a slower passage over a certain region, or if a particular area is to be overflown more than once (say, to observe a seasonal change).

Finally, examination of equations (8) and (9) shows that the <u>direction</u> in which the ascending node will travel around the equator is related to the inclination. Thus $\dot{\Omega} < 0$ if $0 \leq i < 90°$ and $\dot{\Omega} \geq 0$ if $90° \leq i \leq 180°$. Similarly, the direction of periapsis precession can be chosen so that $\dot{\omega} > 0$ if $0 \leq i < 63.4°$ or $116.6° < i \leq 180°$, or $\dot{\omega} < 0$ if $63.4° < i < 116.6°$.

Thus there are 64 possible inclinations which may be obtained from equation (10) by selecting one of two options for each of six parameters -- the location of the ascending node (1) at arrival and (2) at departure; the number of revolutions of (3) the ascending node and of (4) periapsis; and the direction of motion of (5) the ascending node and of (6) periapsis.

The above conditions only serve to determine the orientation of the parking orbit. The shape (or size) of the orbit has to be determined by calculating an eccentricity which satisfies equation (11). Since only nonnegative solutions ($e \geq 0$) can be considered, some of the 64 candidate inclinations may not lead to meaningful solutions.

In summary, for any given approach asymptote and departure asymptote, there are up to 64 discrete parking orbits which satisfy the mission.

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